

Investing for impact

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Env Climate discussion group S14

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2 Main analysis

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Setup (1)

A "social venture" project that

- requires an upfront investment $I > 0$ from an investor
- 1 unit of labor by a manager who
 - allocates unobservable scarce resource between two production technologies (monetary payoff v.s. social benefit), e.g. attention
 - $a_b \geq 0$ = the fraction allocated to social technology
 - $a_x = 1 - a_b \geq 0$ to the for-profit technology
- outcome
 - in the production of the monetary payoff: either "succeed" ($x = X > 0$) or "fail" ($x = 0$).
The probability of a successful outcome: $Pr(x = X | a_x) = f(a_x) \in [0, 1]$, $f' > 0$, $f'' < 0$
 - in the production of social benefit: either "succeed" ($b = 1$) or "fail" ($b = 0$).
The probability of a successful social outcome: $Pr(b = 1 | a_b) = g(a_b) \in [0, 1]$, $g' > 0$, $g'' < 0$

Setup (2)

- 3 agents: profit-motivated investor ($p \implies$ also owner), social impact investor (s), project manager (m)
- If the project is successful in the production of social output, agent $i \in p, s, m$ receives nonpecuniary payoff ψ_i : $\psi_p = 0, \psi_s > 0, \psi_m > 0$
- Agents have different discount rates: $\rho_p = 1, \rho_s = 1 + \delta$ for $\delta > 0$
 - limited capital among socially minded investors
 - additional constraints in the impact investor's portfolio choice problem
 - unmodeled efficiencies coming from for-profit capital providers
- utility function $k_i + \frac{x_i + \psi_i b}{\rho_i}$ where
 - k_i : net-payment made to/from agent i at date 0 (prior to the project)
 - x_i : agent i 's monetary payoff at date 1 depending on $a_x = 1 - a$
 - ψ_i : agent i 's nonpecuniary payoff from a successful social outcome

Assumptions

- Assump. 1: $\lim_{a \rightarrow 1} f'(1 - a) = \infty$, and $\lim_{a \rightarrow 0} g'(a) = \infty$ (unique solution to manager's attention allocation problem given w)
- manager has no wealth and cannot contribute to the upfront investment cost; also cannot receive an upfront transfer as part of her compensation $\implies U_m = x_m + \psi_m b$. Reservation payoff $\gamma \geq 0$ (Assump. 2: γ is sufficiently small)
- x is observable and contractible, but attention is not: assume b is noncontractible
- Limited Liability constraints: no agent receives negative date 1 cash flows + sum of payments to all 3 agents \leq the project's monetary payoff X
 \implies the wage and cash flow sharing rule:
 $\Omega = (w, y, z)$ where manager's equity share $w \geq 0$, impact investor's equity share $y \geq 0$ and the owner's retained equity share $z \geq 0$ with $w + y + z \leq 1$.
- owner's expected payoff is concave in attention (2nd order condition for optimality)

Timeline

- date 0: for-profit owner makes offers
 - to the manager (a wage contract)
 - to the impact investor (a cash flow claim and upfront transfers k_s)
- If manager accepts the wage contract, she chooses $a \in [0,1]$ to maximize her expected payoff:

$$a \in \arg \max_{a' \in [0,1]} E[xw + \psi_m b | a'].$$

Lemma 1 - $a(w)$: higher w , shift more attention to monetary payoff

$$\text{Lemma 2 - } w(a) = a^{-1}(w) = \frac{g'(a)\psi_m}{f'(1-a)X}$$

Date 0

- (a) For-profit owner offers manager wage w
- (b) For-profit owner offers impact investor cash flow claim y at price k_s
- (c) Investors contribute capital I
- (d) Manager chooses attention allocation

Date 1

- (a) x and b realized
- (b) Agents receive specified payments

Benchmark attention a : when there is no impact investor

When the project is solely funded by for-profit investors, social output = a private benefit for the manager:

- A typical principal-agent problem with unobservable effort: choose a wage contract which incentivizes the manager to shift attention away from social output into the for-profit production

$$\max_{\{w,a\}} f(1-a)Xz - I \quad (7)$$

$$\text{s.t. } w = \mathbf{w}(a), \quad (7.1)$$

$$0 \leq f(1-a)Xw + g(a)\psi_m - \gamma, \quad (7.2)$$

$$0 \leq w + z \leq 1, \quad \text{and} \quad w \geq 0, \quad z \geq 0, \quad (7.3)$$

gives $\{w_o, a_o\}, \gamma_o$

Preliminary analysis: with reservation payoff γ_o

For $\gamma < \gamma_o$: (7.2) doesn't bind. FOC of (7) w.r.t. a

$$-f'(1-a_o)X(1-\mathbf{w}(a_o))-f(1-a_o)X\left(\frac{d\mathbf{w}}{da}\bigg|_{a=a_o}\right)=0, \quad (8)$$

For $\gamma \geq \gamma_o$: (7.2) binds.

$$f(1-a_o)X\mathbf{w}(a_o)+g(a_o)\psi_m-\gamma=0. \quad (9)$$

(7.1) must be satisfied $\implies w_o = w(a_o)$

a_o is the lower bound on a ; any optimal contract featuring the involvement of the impact investor achieves an attention allocation as least as large as a_o .

Subsequent analysis: compare equilibrium levels of social attention to the commercial benchmark a_o :

- equilibrium attention $a^* > a_o$: impact investing leads to greater emphasis of social goals
- otherwise: impact investing is ineffective

First-best level of social attention from maximizing joint surplus among the three:

$$-f'(1-a_{FB})X+g'(a_{FB})\left(\frac{\psi_m}{\rho_m}+\frac{\psi_b}{\rho_b}\right)=0, \quad (10)$$

(the model does not provide clear predictions for comparing a^* to a_{FB}

Commitment: optimal contract must be robust to renegotiation

A full-commitment case: for-profit owner must maintain a reputation for achieving sufficient social value.

For-profit owner chooses a contract triplet $\Omega = (w, y, z)$, a corresponding level of social attention a , and a security price k_s to maximize his expected profit.

- Given y and security price k_s , the impact investor's discounted expected payoff is

$$-k_s + \frac{f(1-a)Xy + g(a)\psi_s}{1+\delta},$$

- choose k_s s.t. the expected payoff equal to zero:

$$k_s = \frac{f(1-a)Xy}{1+\delta} + \frac{g(a)\psi_s}{1+\delta}. \quad (11)$$

(discounted expected value of the

impact investor's equity claim y + the impact investor's utility from the social benefit)

Owner's optimal contracting problem

$f(1-a)Xz + k_s - I$:

When the impact investor contributes to the project's upfront investment cost, the for-profit owner's optimal contracting problem is therefore given by,

$$\max_{\{\Omega, a\}} f(1-a)Xz + \frac{f(1-a)Xy}{1+\delta} + \frac{g(a)\psi_s}{1+\delta} - I \quad (12)$$

$$\text{s.t. } w = \mathbf{w}(a), \quad (12.1)$$

$$0 \leq f(1-a)Xw + g(a)\psi_m - \gamma, \quad (12.2)$$

$$0 \leq w + y + z \leq 1, \quad \text{and} \quad \Omega \geq \{0, 0, 0\}, \quad (12.3)$$

Lemma 3 - unique contract maximizing the for-profit owner's expected payoff

$w_v = w(a_v), y_v = 0, z_v = 1 - w_v$:

- $\gamma < \gamma_o : a_v > a_o$
- $\gamma \geq \gamma_o : a_v = a_o$

Full commitment implication

When the for-profit owner can commit not to alter the manager's wage contract, the profit-maximizing contract sets the social impact investor's equity claim to zero. (Any date-1 monetary payment would be inefficient for the impact investor and so y_v is set as small as possible.)

While the impact investor holds none of the project's equity, he still pays for his expected value of social good through the upfront transfer k_s , and it is through this subsidy that the impact investor convinces the for-profit owner to implement a higher level of social attention. \implies a grant or donation!

Limited commitment

meaning: the for-profit owner can renegotiate the manager's wage contract and therefore influence the level of social attention after receiving the payment k_s from the impact investor.
 \Rightarrow the impact investor correctly anticipates the possibility the contract will be renegotiated and adjusts the payment k_s according to the true, rather than promised, level of social attention.

Renegotiate: from $\{(w, y, z), a\} \rightarrow \{(w', y, z'), a\}$ with limited liability ($w' + z' \leq 1 - y$); both for-profit owner and manager better off:

$$E[xz' | a = \mathbf{a}(w')] \geq E[xz | a = \mathbf{a}(w)],$$

$$E[xw' + \psi_m b | a = \mathbf{a}(w')] \geq E[xw + \psi_m b | a = \mathbf{a}(w)].$$

“Renegotiation-proof”: a contract which cannot be renegotiated.

(there cannot be cash burning $w + y + z < 1$; restrict attention to $w + y + z = 1$)

Lemma 4 - renegotiation is only possible by increasing the manager's wage

Lemma 5 - Renegotiation-proof iff $y \geq \underline{y}(a) = 1 - w(a) + \frac{f(1-a)}{f'(1-a)} \frac{dw}{da}$, unique threshold

Owner's optimal contracting problem

With limited commitment, the for-profit owner's optimal contracting problem is given by

$$\max_{\{\Omega, a\}} f(1-a)Xz + \frac{f(1-a)Xy}{1+\delta} + \frac{g(a)\psi_s}{1+\delta} - I \quad (14)$$

$$\text{s.t. } w = \mathbf{w}(a), \quad (14.1)$$

$$0 \leq f(1-a)Xw + g(a)\psi_m - \gamma, \quad (14.2)$$

$$1 = w + y + z, \quad \text{and} \quad \Omega \geq (0, 0, 0), \quad (14.3)$$

$$y \geq \underline{y}(a), \quad (14.4)$$

where we have again substituted for the upfront transfer k_s from (11) into the for-profit owner's expected profit in (14) and where constraint (14.4) ensures that the impact investor's equity claim is large enough so that the contract is renegotiation-proof.

Propositions

Proposition 1. There exists a unique optimal contract $\{(w_r, y_r, z_r), a_r\}$ which maximizes the for-profit owner's expected payoff and which is renegation-proof. The contract is characterized by $w_r = \mathbf{w}(a_r)$, $y_r = \underline{\mathbf{y}}(a_r)$, and $z_r = 1 - w_r - y_r$.

1. For $\psi_s > \underline{\psi}_s$ (and $\gamma < \gamma_o$), the social impact investor holds a nonzero equity claim, $y_r > 0$, and social attention exceeds the benchmark level, $a_r > a_o$. The unique threshold $\underline{\psi}_s$ is given by

$$\underline{\psi}_s = \frac{\delta f(1 - a_o) X \left. \frac{dy}{da} \right|_{a=a_o}}{g'(a_o)}. \quad (15)$$

2. Otherwise, $y_r = 0$ and $a_r = a_o$.

summarized by Figure 3 in paper)

Corollary 1

Proposition 1 shows that impact investors hold cash flow claims only in those firms with significant social value.

Corollary 1. If the manager's participation constraint is slack and $\psi_s > \underline{\psi}_s$,

$$\frac{dy_r}{d\psi_s} > 0 \quad \text{and} \quad \frac{dy_r}{d\delta} < 0.$$

Corollary 1 shows that:

- impact investors hold larger equity stakes in more socially valuable firms. (size of upfront subsidy depends on the impact investor's expected social value)
- the impact investor's equity stake is decreasing in the impact investor's added cost of capital δ .

Impact investing is optimal only for those projects for which the monetary payoff and social benefit are truly coupled.

Non-profit firm = $z = 0$, when the for-profit owner retains none of the project's residual cash flow

Collary 2 - Nonprofit status is never optimal.

From Proposition 1, the impact investor's equity stake is given by $y_r = 1 - w(a_r) + \frac{f(1-a_r)}{f'(1-a_r)} \frac{dw}{da} \Big|_{a=a_r}$, which is the smallest financial claim needed to dissuade the for-profit owner from renegeing on the promised level of social attention a_r . Since $\frac{dw}{da} < 0$ for all $a \in (0, 1)$, the impact investor's equity stake is strictly less than the entire residual cash flow claim, and the for-profit owner therefore retains a positive equity stake, $z = -\frac{f(1-a_r)}{f'(1-a_r)} \frac{dw}{da} \Big|_{a=a_r} > 0$.

To see this: when committed, the change in the for-profit owner's expected date 1 payoff from a decrease in the level of social attention cannot be positive

$$\underbrace{f'(1-a)X(1-w(a)-y)}_{+} + \underbrace{f(1-a)X\left(\frac{dw}{da}\right)}_{-} \leq 0. \quad (16)$$

Corporate taxes

If there are additional benefits to nonprofit status, such as tax advantages, nonprofit firms may be optimal when the desired level of social attention is sufficiently high.

With tax: the for-profit owner's expected payoff is equal to the value of his expected after-tax equity claim plus the value of the upfront payment from the impact investor (which reflects the tax paid on the impact investor's equity claim) minus the cost of investment

$$f(1-a_r)X\left(1-\mathbf{w}(a_r)-\underline{\mathbf{y}}(a_r)\right)(1-\tau) + \frac{f(1-a_r)X\underline{\mathbf{y}}(a_r)(1-\tau)+g(a_r)\psi_s}{1+\delta} - I. \quad (17)$$

VS. the entire after wage cash flow is assigned to the impact investor, and these profits are not subject to corporate taxation

$$\frac{f(1-a_r)X(1-\mathbf{w}(a_r))+g(a_r)\psi_s}{1+\delta} - I. \quad (18)$$

Result

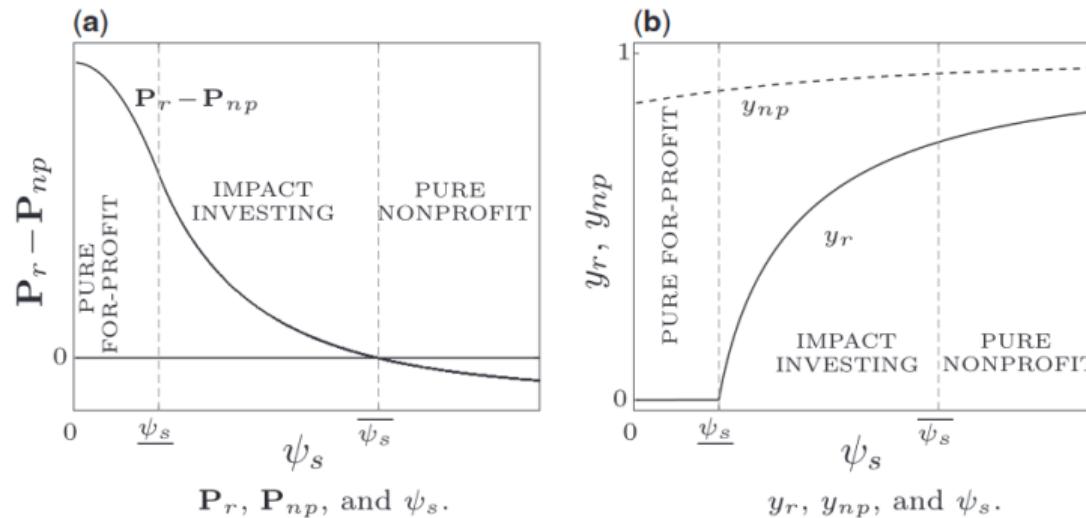


Figure 5

A comparison of impact investing to nonprofit status

The first panel depicts the difference in the for-profit owner's profit under impact investing (P_r) and under nonprofit status (P_{np}) as a function of ψ_s . The second panel depicts the impact investor's cash flow claims under impact investing (y_r) and under nonprofit status (y_{np}) as a function of ψ_s .

The end!