Introduction

Questions :

* in an equilibrium asset pricing model with production & public good provision

- What effect investors have on social outcomes ?
  - > Public goods provision = mitigation of climate change
  - > consequentialist investors : capital allocation between production and investment in mitigating climate change
- If effective, how does it compare to (and interact with)

government public goods provisions

→ taxes

→ subsidies

in achieving impact ?

Preview of results

- Environmentally conscious investors: (v.s. financial investors)
  - invest more wealth
    ∵ public good preference = less risk averse
  - invest more in clean firms
    when corr(clean, dirty) is low, also invest more in dirty
    ∵ hedging
  - prices depend:
    premium + negative alpha if env. contribution > sys. risk
    Implication: if a clean firm carries ++ sys. risks,
    ⇓ outperform (exp. return ↑)
- Free-riding effect: underprovision relative to SP's choice
  ∵ investors do not internalize the benefits of their investments on others

Extensions

- dirty firm contributes negatively to the public goods
  [negative externality]
  ⇒ env. investors invest less in the dirty firms
  ⇒ total provision ↑ ∵ investments concentrate on clean firms

- allow direct provision (donations)
  (giving up financial returns in exchange for more public good and less portfolio risk)
  weakly increase total provision

- uncertainty in how public good contributions result in benefits
  ⇒ uncertainty ↑ = { public good production ↓ ; green investment ↓ }
  ⇕
  corr (clean, dirty) ↑

Model

2 periods

t=0:

- companies sell shares to investors
- investors invest in these shares and riskless assets
- Companies allocate capital to their normal production and to climate change mitigation

t=1:

- production occurs
- State of world realizes
- Investors enjoy payoff

Investors

Environmental conscious investor E and financial investor F

Both investors start with an endowment of initial wealth W_0 at $t=0$

i

- mean variance utility from terminal wealth ($\tilde{W}_i$, stochastic)
  + additional term $f(G)$: value from public good consumption

amount of public goods

$f'(\cdot) > 0, \quad f''(\cdot) \leq 0$

$$U_i = E_0(\tilde{W}_i) - \nu \, Var(\tilde{W}_i) / (2 W_0) + \psi_i f(G)$$

risk aversion

how much investor cares about G

$\psi_E > \psi_F$

$t=0$: investor i buys shares using W_0 ⇐ each firm's stock in positive net supply

trade a riskless asset ⇐ zero net supply

$t=1$: state of world realized ⇒ terminal wealth $\tilde{W}_i$ and G determined

Budget Constraints

- stochastic return of firm n = r_n

  $\tilde{r}_n = r_n - r_f$ excessive return

- Let $\theta_{i,n}$ be the fraction of investor i's wealth invested in firm n and $\theta_{i,f}$ be the fraction of her wealth invested in the risk-free asset. Her budget constraints at $t = 0$ and $t = 1$ are as follows:

dirty firm D
clean firm C

$$t=0 \qquad \sum_{n\in(C,D)} \underbrace{\theta_{i,n}\omega_0}_{\text{risky}} + \underbrace{\theta_{i,f}\omega_0}_{\text{risk-free}} = \omega_0,$$

$$t=1: \qquad \tilde{\omega}_i = \sum_{n\in(C,D)} (1 + r_n)\theta_{i,n}\omega_0 + (1 + r_f)\theta_{i,f}\omega_0.$$

$$\Rightarrow \qquad \tilde{\omega}_i = \omega_0 \left(1 + r_f + \sum_{n\in(C,D)} \tilde{r}_n\theta_{i,n} \right), \tag{1}$$

investor E's problem (sophisticated)

$$\underbrace{\max}_{\Theta_E = (\theta_C,\, \theta_D)'} U_E = \mathbb{E}_0(\tilde{\omega}_E) - \nu Var(\tilde{\omega}_E)/(2\omega_0) + \psi_E f(G),$$

where $\tilde{\omega}_E = \omega_0 \left(1 + r_f + \sum_{n \in (C,D)} \tilde{r}_n \Theta_E\right)$ and $G = \underbrace{H(\Theta_E, \Theta_F)}_{\text{internalize the effect of their own portfolio}}$.

Firm's problem: Production of private goods

productivity shock

trade-off

$$\tilde{y}_n = \tilde{\epsilon}_n (k_n - z_n)^{\gamma_n} z_n^{1-\gamma_n},$$

total investment

amount dedicated to public goods

$\gamma_C < \gamma_D \leq 1$

$\gamma_D = 1$ means public good production is only costly for D (without benefits)

Production of public goods: $G = \sum_n z_n$

Excessive financial return $\tilde{r}_n = \dfrac{\tilde{y}_n}{k_n} - 1 - r_f$

endogenously determined in equilibrium

$\tilde{\varepsilon}_n \sim N(0, \sigma_\varepsilon^2) \implies \tilde{r}_n \sim N(\bar{r}_n, \sigma^2)$

Firms maximize contingent claim value for shareholders

Firm n's maximization decision is given by:

stochastic discount factor

$$\underbrace{\max}_{z_n, k_n} E_0 \tilde{M} \tilde{\epsilon}_n (k_n - z_n)^{\gamma_n} z_n^{1-\gamma_n} - k_n,$$

and firm n's investment decisions yield (see proof in Appendix A.1):

$$z_n = (1 - \gamma_n) k_n. \tag{2}$$

$$\Longrightarrow \quad \tilde{y}_n = \tilde{A}_n k_n, \text{ where } \tilde{A}_n = \tilde{\epsilon}_n \gamma_n^{\gamma_n} (1 - \gamma_n)^{1-\gamma_n}.$$

z_n is determined by $\begin{cases} k_n & \leftarrow \text{determined in equ.} \\ \gamma_n & \text{exogenous} \end{cases}$

3 Equilibrium

We now solve for the equilibrium allocation. The equilibrium consists of the investors' portfolio allocations (Θ_E, Θ_F), firm investments (k_C, k_D, z_C, z_D), and stock expected excess returns $\bar{r}_C, \bar{r}_D$ such that:

(i) the investors' portfolios maximize their expected utility;

(ii) each firm chooses investments to maximize contingent claim value for shareholders;

(iii) asset markets clear;

$$k_n = \sum_i \theta_{i,n}\omega_0, \tag{3}$$

$$\sum_i \theta_{i,f} = 0, \tag{4}$$

and payoffs are realized and distributed, and the public good is produced $G = \sum_n z_n$.

$$\eta_{i,n} = 1 - \gamma_n, \tag{5}$$

$$\bar{r} = \nu \Sigma \Theta - (1-\gamma)\psi f'(G), \tag{6}$$

Proposition 1. *The optimality conditions for investor i, $i \in \{E, F\}$, lead to the following asset allocation:*

$$\theta_{i,C} = \frac{\Big(1-\gamma_C - \rho(1-\gamma_D)\Big)\psi_i f'(G) + \bar{r}_C - \bar{r}_D \rho}{\nu(1-\rho^2)\sigma^2},$$
$$\theta_{i,D} = \frac{\Big(1-\gamma_D - \rho(1-\gamma_C)\Big)\psi_i f'(G) + \bar{r}_D - \bar{r}_C \rho}{\nu(1-\rho^2)\sigma^2} \tag{7}$$

① $\left.\begin{array}{l}\psi_E > \psi_F \\ \gamma_C < \gamma_D\end{array}\right\} \Rightarrow \theta_{E,C} > \theta_{E,D}$

investor E invests more in the clean firm than investor F, i.e. $\theta_{E,C} > \theta_{F,C}$.

② $\rho > \dfrac{1-\gamma_D}{1-\gamma_C} \Rightarrow \theta_{ED} < \theta_{FD}$

correlation between returns is large

prop. 1 : $\theta_{E,f} < 0$

$\{ \theta_{D,f} > 0$

Corollary 1. *In equilibrium, investor E shorts the riskless asset and investor F longs it. Investor E invests less (more) in the dirty firm than investor F when the stock correlation (ρ) is larger (smaller) than the relative weighting of the dirty and clean firm on the public good ($\frac{1-\gamma_D}{1-\gamma_C}$).*

$\Downarrow$ investor E borrows from investor D at the risk-free rate

Lemma 1. *The absolute risk aversion of environmentally conscious investors is lower than that of financial investors.*

Asset pricing implications

premultiply by M

market equil.

$$\bar{r} = \nu\Sigma\Theta - (1-\gamma)\psi f'(G), \quad (6) \iff \bar{r}_M = \Theta'\bar{r}$$

$\tilde{r}_n \sim N(\bar{r}, \sigma^2)$

$$\Longrightarrow \quad \bar{r}_M = \nu\sigma_M^2 \boxed{- (1 - \sum_n \theta_n\gamma_n)\psi f'(G).} \quad (8)$$

Proposition 2. *The expected excess returns of firm n in equilibrium are:* β_n

$$\bar{r}_n = \beta_n^G \bar{r}_M, = \beta_n \bar{r}_M + \alpha_n \quad (9)$$

where

$$\boxed{\beta_n^G = \frac{Cov(\tilde{r}_M, \tilde{r}_n) - \nu^{-1}(1-\gamma_n)\psi f'(G)}{\sigma_M^2 - \nu^{-1}(1 - \sum_n \theta_n\gamma_n)\psi f'(G)}.} \quad (10)$$

Let β_n be the standard CAPM beta, i.e., $\beta_n = \frac{Cov(\tilde{r}_M, \tilde{r}_n)}{\sigma_M^2}$.

$$\Longrightarrow \quad \bar{r}_n = \frac{Cov(\tilde{r}_M, \tilde{r}_n)}{\sigma_M^2}\bar{r}_M + \alpha_n = \beta_n \bar{r}_M + \alpha_n, \tag{11}$$

public good factor
not captured by traditional CAPM

$$\Longleftrightarrow \quad \exists \text{ abnormal return } \alpha_n \textit{ iff } \beta_n \neq \beta_n^G.$$

Given $\bar{r}_m > 0$

$$\beta_n < \frac{1 - \gamma_n}{1 - \sum_n \theta_n \gamma_n}. \quad = \eta$$

If condition (12) holds, then $\beta_n^G < \beta_n$ and therefore $\alpha_n < 0$. If condition (12) does not hold, then $\beta_n^G \geq \beta_n$ and therefore $\alpha_n \geq 0$.

Proposition 3. *Whether firm n generates CAPM-alpha depends on its systematic risk β_n and its relative public good investment contribution, i.e.,* $\frac{1-\gamma_n}{1-\sum_n \theta_n \gamma_n}$.

1. *When $\beta_C \leq 1$, the clean firm generates a negative alpha and the dirty firm generates a positive alpha.*

2. *When $\beta_C > 1$, the sign of alpha for each firm depends on the parameters.*

As shown in the proof, the necessary and sufficient condition for $\beta_C \leq 1$ to hold is $\theta_C \leq \theta_D$. Furthermore, whenever $\beta_C \leq 1$, $\beta_D \geq 1$. This means that when clean firms are a smaller part of the market portfolio than dirty firms, clean firms exhibit a systematic risk lower than 1 and dirty firms exhibit a systematic risk higher than 1. Since clean firms' relative public

Social Planner's solution (P19)

$$\max_{\{\theta^p_{i,n}\}} \frac{1}{2}\mathbb{E}_0\tilde{\omega}_E + \frac{1}{2}\mathbb{E}_0\tilde{\omega}_F - \frac{\nu}{4\omega_0}\left[Var(\tilde{\omega}_E) + Var(\tilde{\omega}_F)\right] + \frac{1}{2}(\psi_E + \psi_F)f(G),$$

$$s.t. \quad \tilde{\omega}_E = \omega_0\left(1 + r_f + \sum_{n\in(C,D)} \tilde{r}_n\theta^p_{E,n}\right), \tag{13}$$

$$\tilde{\omega}_F = \omega_0\left(1 + r_f + \sum_{n\in(C,D)} \tilde{r}_n\theta^p_{F,n}\right), \tag{14}$$

$$G = H(\Theta^p_E, \Theta^p_F), \tag{15}$$

Propo. 4:

$$\theta^p_{i,C} = \frac{\Big(1-\gamma_C-\rho(1-\gamma_D)\Big)\sum_i\psi_i f'(G^p) + \bar{r}^p_C - \bar{r}^p_D\rho}{\nu(1-\rho^2)\sigma^2},$$

$$\theta^p_{i,D} = \frac{\Big(1-\gamma_D-\rho(1-\gamma_C)\Big)\sum_i\psi_i f'(G^p) + \bar{r}^p_D - \bar{r}^p_C\rho}{\nu(1-\rho^2)\sigma^2} \tag{16}$$

Investors have identical asset allocations.

Proposition 5. *In the planner's equilibrium, the expected excess returns of firm n are expressed as:*

$$\bar{r}_n^p = \beta_n^{G^p} \bar{r}_M^p, \tag{17}$$

where

$$\beta_n^{G^p} = \frac{Cov(\tilde{r}_M^p, \tilde{r}_n^p) - 2\nu^{-1}(1-\gamma_n)\psi f'(G^p)}{\sigma_M^2 - 2\nu^{-1}(1-\sum_n \theta_n^p \gamma_n)\psi f'(G^p)}. \tag{18}$$

Controlling for $\bar{r}$, the planner's equilibrium produces more public goods than the market equilibrium.

Taxation & Crowding out

When the government levies tax τ from each investor, there is 2τ available to contribute towards G. To capture the inefficiency or any organizational costs in the government (Bandiera et al., 2009), we model a waste cost of λ, namely, for τ amount of taxes levied, only $(1-\lambda)\tau$ effectively contribute to public good provision.[18]

With taxes, the total amount of public good provision amounts to:

$$G(\tau) = 2\Big(\tau(1-\lambda) + (\omega_0-\tau)\theta_C(\tau)(1-\gamma_C) + (\omega_0-\tau)\theta_D(\tau)(1-\gamma_D)\Big). \quad (19)$$

allocation as a function of cost λ
but only through G!

Since investors take taxes as given and the tax is lump-sum, in this environment the only way taxes could affect the investors' portfolio choices is through the public good G (recall from Proposition 1 that $f'(G)$ affects investors' portfolio choices).[19]

τ $f'(G)$

Lemma 2. *When $f''(G) = 0$, levying τ from investors' initial wealth does not alter asset prices nor allocations.*

Proposition 6. *Suppose $f''(G) = 0$. When the government levies taxes τ from each investor, it crowds out private provision of public goods by $2\tau(\lambda + \sum_n (1 - \gamma_n)\theta_n)$. Therefore:*

1. *If $\lambda \leq \gamma_C$, the total provision of public goods always increases; hence, crowding out is not complete.*
2. *If $\lambda > \gamma_C$, whether total provision of public goods increases depends on the relative strength between private wealth allocation to the clean firm and the government's waste cost.*
   - (a) *If $\theta_C < \frac{\gamma_D - \lambda}{\gamma_D - \gamma_C}$, crowding out is not complete - the total provision of public goods increases.*
   - (b) *If $\theta_C > \frac{\gamma_D - \lambda}{\gamma_D - \gamma_C}$, the total provision of public goods decreases.*

Green Subsidy = firm receives δz for a green investment z

$\hookrightarrow z = z(\delta)$

funded by taxes

\$1 tax

$\Rightarrow$ \$ $1-\lambda$ available for subsidy

$\Rightarrow$ changes $\bar{r}_n(\delta)$

Proposition 7. *In equilibrium,*

1. *Firm n's green investment z_n is:*

$$z_n = \frac{1-\gamma_n}{1-\delta} k_n. \tag{24}$$

2. *For any level of green subsidy $\delta \in (0,1)$, $\bar{r}_n(\delta) > \bar{r}_n$, and $\frac{d(\frac{1+\bar{r}_n(\delta)}{1+\bar{r}_n})}{d\gamma_n} < 0$. A green subsidy does not change the asset pricing implications of condition (12) for CAPM alphas.*

3. *The total amount of public good provided has a non-linear relationship with the green subsidy, and*

$$G = \frac{\omega_0}{\Xi + \Phi},$$

where $\Xi \equiv \frac{1-\delta}{2(1-\gamma_D+(\gamma_D-\gamma_C)\theta_C)}$ and $\Phi \equiv \frac{\delta}{2(1-\lambda)}$. Note that $\frac{d\Phi}{d\delta} > 0$ and whenever the semi-elasticity $\epsilon_{\theta_C,\delta} > (\delta-1)^{-1}$, $\frac{d\Xi}{d\delta} < 0$.